Solutions

Math 2D Quiz 4 Afternoon - February 11th

Please put name and ID on *both* sides for grading and redistribution!

Show all of your work. *There is a question on the back side.

1. Prove whether or not the following limit exists:

$$\lim_{(x,y)\to(0,0)} \frac{x^2\tan^2(y)}{x^4+y^4}.$$

You are allowed to use polar coordinates, but it is not necessary. It might actually be more difficult.

(The $\tan^2 y \approx y$ when $y \approx 0$ so we have "equal powers" of r) in top and bottom.

↳ Thus, we expect it to NOT Exist.

One path: let $x = 0$, $\lim_{(0,y)\to(0,0)} \frac{0\cdot\tan^2 y}{0+y^4} = \boxed{0}$ +1

Another Path: let $x = my$ $\lim_{y\to 0} \frac{m^2y^2\tan^2 y}{y^4(1+m^4)} = \frac{m^2}{(1+m^4)} \lim_{y\to 0} \frac{\tan^2 y}{y^2}$. +1

[You may note $\lim_{y\to 0} \frac{\tan^2 y}{y^2} = \lim_{y\to 0} \frac{1}{\cos^2(y)} \cdot \frac{\sin^2 y}{y^2} = 1$]

[Otherwise, $\overset{\text{L'Hopital}}{=} \frac{m^2}{(1+m^4)} \lim_{y\to 0} \frac{2\tan y \sec^2 y}{2y}$; $\sec(0) = 1$ so we need $\lim_{y\to 0} \frac{\tan y}{y}$

+3 $\overset{\text{L'Hopital}}{=} \frac{m^2}{(1+m^4)} \lim_{y\to 0} \frac{\sec^2 y}{1} = \frac{m^2}{(1+m^4)} \cdot 1$]

Regardless, on another path, it limits to $\frac{m^2}{(1+m^4)}$ // NOT zero.

Thus, limit does NOT exist.

1 pt each

2. Let $G(x, y) = \ln(x^2 + y^2)$. You will be computing all sorts of derivatives.
(a) Compute the first partials: G_x and G_y.
(b) Compute the second partials: G_{xx}, G_{yy}, and G_{xy}.
For this function, $G_{yx} = G_{xy}$ so you may compute G_{yx} instead of G_{xy} if you wish.
(c) **Bonus** Determine if G satisfies $\Delta G = G_{xx} + G_{yy} = 0$.
This time, if you are missing any credit on the quiz at all, part (c) is 1 extra credit point.

a) $G_x(x,y) = \dfrac{2x}{x^2+y^2}$; $G_y(x,y) = \dfrac{2y}{x^2+y^2}$

b) $G_{xx}(x,y) = \dfrac{2(x^2+y^2) - 2x(2x)}{(x^2+y^2)^2} = \dfrac{2y^2-2x^2}{(x^2+y^2)^2}$

$G_{yy}(x,y) = \dfrac{2(x^2+y^2) - 2y(2y)}{(x^2+y^2)^2} = \dfrac{2x^2-2y^2}{(x^2+y^2)^2}$

$G_{xy}(x,y) = - \dfrac{4xy}{(x^2+y^2)^2}$

Alternatively, $G_{yx}(x,y) = \dfrac{0 - 2y \cdot 2x}{(x^2+y^2)^2} = - \dfrac{4xy}{(x^2+y^2)^2}$

c) $G_{xx} + G_{yy} = \dfrac{\cancel{2y^2} - \cancel{2x^2} + \cancel{2x^2} - \cancel{2y^2}}{(x^2+y^2)^2} = 0$

(Yes it satisfies $\Delta G = 0$)